- Que. What is Zone - plate?
Explain construction of a zone - plate with its theory. Explain how a zone plate acts like a convergent and divergent lens?
Derive an expression for its focal length?

"OR"

Explain, Zone-plate acts like a convergent lens having multiple foci. Derive an expression for its focal length.

"OR"

What is a zone-plate? How is it constructed? Give its theory and show that it has multiple focii. Compaire its action with that of a convex lens.

Ans Zone - plate :→
It is transparent plate on which circle whose radii

are proportional to the square roots
8 of natural number 1, 2, 3 ...... are
drawn. the alternate number zones
9 thus formed are blackened. such a
plate behaves like a convex lenses
10 and produces an images of a source
of light on the screen placed
11 of at a suitable distance.
In other words zone-
12 plate is a specially constructed
screen such that light is obstructed
13 from every alternate.

Construction → A zone-plate is

Sunday

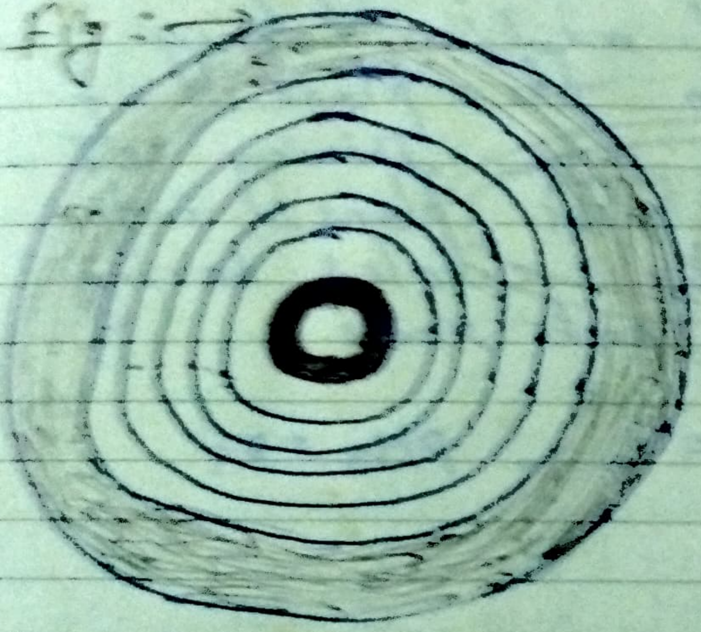
March 3 62nd Day

Constructed by drawing a system of
Conectric circle on a shift while
Cardboard. whose radii are propotional
to the square roots of the natural
number. Alternate zones are then
blackned. and a much reduced photor
graph of this drawing is obtained
on a glass plate. two such zone

Notations

plate are showing A(a) and A(b) when the central zone is clear, the zone plate is said to be positive and if the central zone is opaque is called negative. the negative zone plate when held in the path of light from a distant point source.

Fig :—



(a)

(b)

Produce a large intensity at a point on its axis at a distance corresponding to the size of the zone and the wave length of light used.

Theory :→ Let O be a luminous point object emitting spherical waves of wave length λ whose effect at the point I on the screen is required. Let us consider an imaginary plane through P of a transparent medium lying perpendicular to the plane of paper and the line joining OI. Let us divide this plane into zones bounded by circles having centres at P and radii.

$PM_1 = r_1$, $PM_2 = r_2$, $PM_3 = r_3$

$PM_n = r_n$.

Such that,

$$OM_1 + M_1I = OP + PI + \lambda/2$$

$$OM_2 + M_2I = OP + PI + 2\lambda/2$$

$$OM_3 + M_3I = OP + PI + 3\lambda/2$$

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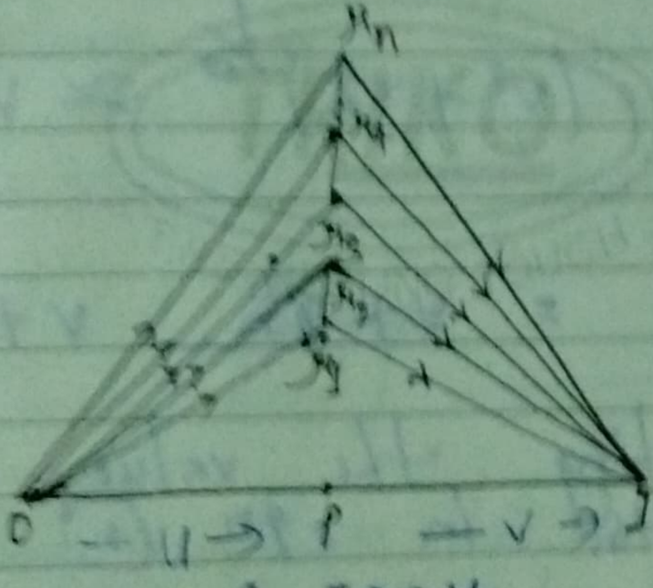
$$OM_n + M_nI = OP + PI + n\lambda/2$$

The angular rings thus formed are half period zones, for the images, because the length of the path of the light through the corresponding point of any two consecutive zones differs by $\lambda/2$.

To find the radius r_n of the nth circle, we have,

$$OM_n + IM_n = OP + IP + n\frac{\lambda}{2} \longrightarrow (1)$$



Let, $OP = u$ and $IP = v$

$$OM_n^2 = OP^2 + PM_n^2 = u^2 + r_n^2$$

or,

$$OM_n = u\left[1 + \frac{r_n^2}{u^2}\right]^{1/2}$$

$$= u\left[1 + \frac{r_n^2}{2u^2}\right] = \left(u + \frac{u r_n^2}{2u^2}\right)$$

$$OM_n = u + \frac{r_n^2}{2u}$$

Because u is very very large in comparism with r_n and hence higher lowers of $\frac{r_n^2}{u^2}$ can be neglected.

Again,

$$IM_n^2 = PI^2 + PM_n^2 = v^2 + r_n^2$$

$$\therefore IM_n = (v^2 + r_n^2)^{1/2} \simeq v\left(1 + \frac{r_n^2}{2v^2}\right)$$

$$= \cancel{v + r_n^2} \quad v + \frac{r_n^2}{2v}$$

Substituting the value of OM_n and IM_n in eqn (1) we get.

$$u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} = u + v + \frac{n\lambda}{2}$$

OR, $$\frac{r_n^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right) = \frac{n\lambda}{2}$$

or, $$r_n^2\left(\frac{1}{u}+\frac{1}{v}\right)=n\lambda \quad \text{---(ii)}$$

or, $$r_n^2=\frac{n\lambda uv}{u+v} \quad \text{---(iii)}$$

Since u and v and λ are constant
$r_n^2 \propto n$

$$\therefore\ r_n \propto \sqrt{n} \quad \text{---(iv)}$$

Now, the area of the n^{th} zone is given by,

$$\pi\left(r_n^2 - r_{n-1}^2\right)=\pi\left\{\frac{n\lambda uv}{u+v}-\frac{(n-1)\lambda uv}{u+v}\right\}$$

$$=\pi\lambda uv \quad \text{---(v)}$$

This show that the area of the n^{th} zone is independent of n. further's the area diminishes as u or, v decrease i.e. as the plate is nearer to object or, images further, as the wave from successive transparent zones differ in both λ, the wave from than reach I in the same phase.

Thus, if $A_1, A_2, A_3 \ldots A_n$ be the amplitudes due to I 1st, 2nd, 3rd ---- and nth zone respectively the resultant amplitude at I will be,

$$A = A_1 + A_3 + A_5 ------ + A_n$$

which is much greater than $\frac{A_1}{2}$ the resultant amplitude. If all the zones were transparent. Hence the point I is extremely

bright and can be said to be the images of O, and when the whole wave front is unobstructed the amplitude is given by,

$$A = A_1 - A_2 + A_3 - A_4 ---- + A_n$$

$$= \frac{A_1}{2}$$ (if n is very carry and ni odd).

Notations

This explains the focussing action of a zone plate, as it focuses the light from O at I. Thus, a zone plate behaves like a convex lens (converging lens).

Focal - Length :→

Explanation of zone plate acts like a convergent lens having multiple foci.

we know that from equation (11),

$$r_n^2\left(\frac{1}{u}+\frac{1}{v}\right)=n\lambda$$

we have,

$$\boxed{\frac{1}{u}+\frac{1}{v}=\frac{n\lambda}{r_n^2}} \quad \text{—— (vi)}$$

This is result similar to the one found for a convex lens of focal length f. i.e;

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad \text{—— (vii)}$$

$$\therefore \frac{1}{f}=\frac{n\lambda}{r_n^2} \quad \text{from eq}^n$$

or, $f = \frac{r_n^2}{n\lambda}$

This gives the focal length of the zone-plate. If the radius of the half period zone is 0.316 mm and the wave length of light $\lambda = 5 \times 10^{-5}$ cm than the zone plate will be have like a convergent lense of focal length 20 cms.

Thus we find that a zone plate behaves like convergent lense.